Cobordisms, global symmetries & quantum gravity

We only consider theories in $d \geq 4$ in general.

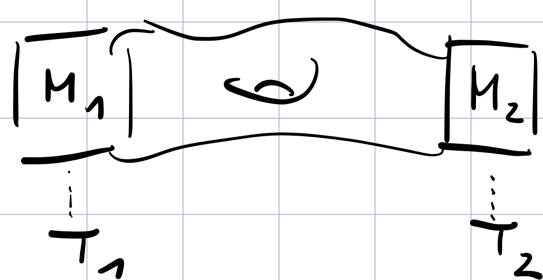
Physics:
* predicts new defects to trivialize theories
* No α-parameters in QG

Mathematics:
Cobordism groups

Cobordism Conjecture:
(McNamara, Vafa, '19)

QG ($d \geq 4$) is unique: Any 2 such QG theories must be deformable into each other ("cobordant").

(−1) form
$d\alpha_\lambda = 0$
$\|$
$\lambda \int \alpha_\lambda$
$\to \int \phi(x)\alpha_\lambda$ (gauging)



↔ absence of global 1-form symmetries (incl. non-invert.)

Completeness Conjectures
→ no irrational charges

No free parameters in QG

(Asymptotic) WGC

$\boxed{\text{No global symmetry Conjecture}}$

→ Compactness of symm. groups

Every symmetry in QG ($d \geq 4$) either
* broken ⟷ through QG induced effect, e.g. gravitational $e^{-S_{grav.}}$
* gauged ⟷ e.g.: a $U(1)$ is actually

sum over principal G-bundles in path-integral! (both cobordisms &

discrete) a massive U(1) gauge symmetry

No global symmetries :
aspires to apply to all types of symmetries
* 0-form symmetries : continuous & discrete
* higher p-form
* non-invertible / categorical

Evidence / Support:

i) Top-down: perturbative string theory — most directly: 0-form continuous

ii) Bottom-up: Black Hole arguments

iii) Special case of AdS gravity: Holography — also generalised / discrete

Literature: Reviews: Palti: 1903.06239
Vafa et al. 2212.06187

Banks-Dixon: '88
Banks-Seiberg: 1011.5120
Heidenreich et al: 2104.07036
⋮

i) Evidence from pert. string theory [Banks, Dixon '88]

Idea: Global continuous symmetry G in spacetime $\mathcal{M}$

$\downarrow$ 1

affine G current algebra on string WS Σ

$\downarrow$ 2

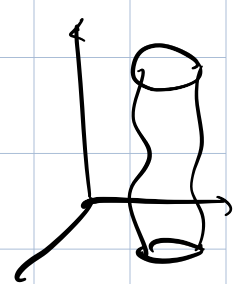
Gauge symmetry in target space $\mathcal{M}$

ad 1 : Consider exact global continuous symmetry acting on physical states of pert. string theory

$\longrightarrow$ global symmetry of SCFT on Σ

Note: Not applicable to Kalb-Ramond & Ramond-Ramond higher form symm. (do not couple to pert. string states)

a) Assume first: $\partial\Sigma = 0$



Noether current $j^a_\alpha(z,\bar{z})$:

$$\partial^\alpha j^a_\alpha(z,\bar{z}) = 0 \qquad a = 1, \dots, \dim(G)$$

w/ symmetry generator:

$$Q^a = \oint dz \; j_z^a(z,\bar{z}) + \oint d\bar{z} \; \bar{J}_{\bar{z}}^a(z,\bar{z})$$

BRST invariance of Q^a $\longrightarrow$ Q^a commutes with superconformal generators on Σ

$j^a(z,\bar{z})$: $(h,\bar{h}) = (1,0)$

$\bar{J}^a(z,\bar{z})$: $(0,1)$

In <u>compact</u> CFT w/ positive metric this can be shown to give:

$$\bar{\partial} j^a = 0, \quad \partial \bar{J}^a = 0$$

$$j^a(z) = \sum_{n\in\mathbb{Z}} j^a_n \, z^{n+1} : \quad [j^a_n, j^b_m] = i f^{ab}{}_c \, j^c_{m+n} + n \cdot k^{ab} \delta_{n+m,0}$$

$\rightarrow$ affine / Kac – Moody algebra

b) If $\partial\Sigma \neq 0$: Symmetry can also act on boundary of Σ only

$\rightarrow$ $j^a(z) \longrightarrow \lambda^a$ constant

(2) Can construct vertex operator

$$V = \left\{ \begin{matrix} j^a(z) \\ \lambda^a \end{matrix} \right\} \tilde{\psi}^\mu(\bar{z})\, e^{ikX(z,\bar{z})}$$ of massless vector in adjoint of G

$\longrightarrow$ symmetry gauged in $\mathcal{H}$ in spacetime $\mathcal{H}$

Examples:

- heterotic $E_8 \times E_8$ / $SO(32)$:

$$j^a(z) = \lambda^A(z)\, \lambda^B(z)\, t^a_{AB}$$

- circle compactification of superstring of direction X_a

$$j^a(z) = \psi^a(z)$$

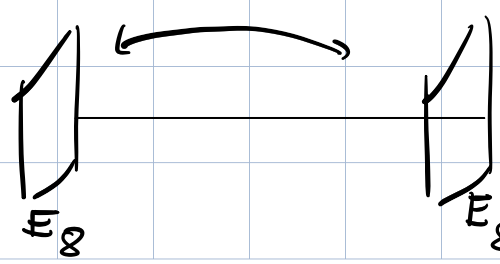
Furthermore:

By inspection, known discrete symmetries are in fact "gauge symmetries" (at least in UV)

E.g.: a) $E_8 \times E_8 \times \mathbb{Z}_2$ heterotic

$\downarrow$

Parity of x^{11} in M-theory on $I \times \mathbb{R}^{1,9}$



$\rightarrow$ part of diffeomorphism group

b) $SL(2,\mathbb{Z})$ of IIB

= gauge symmetry = redundancy of description

equivalently:

IIB on $\mathbb{R}^{1,8} \times S^1 \cong$ M-theory on T^2

$SL(2,\mathbb{Z})$ = isometry of T^2

More generally: Compactification of QG on $\mathcal{M}$ w/ isometry G

$\longrightarrow$ gauged symmetry G

c) Discrete $\mathbb{Z}_k$ symmetry acting on pert. string states

are remnants of

"Stückelberg" massive $U(1)$ gauge symmetries:

A: $U(1)$ gauge field

$\phi(x)$: scalar field w/ shift $\phi(x) \cong \phi(x) + 2\pi$

$$\mathcal{L} = t^2 \cdot \int (d\phi + kA) \wedge * (d\phi + kA) + \int \mathcal{L}(A)$$

invariant under combined

$$\phi(x) \to \phi(x) + k\lambda(x)$$
$$A(x) \to A(x) - \lambda(x)$$

$\phi(x)$ becomes component of massive $U(1)$ gauge field

$$\mathcal{L} \supset t^2 \cdot k^2 \cdot \int A \wedge * A$$

At $E \ll t^2 k^2$: $U(1)$ remains as perturbative global $U(1)$

but gravitational instantons break $U(1) \to \mathbb{Z}_k$:

$$S \supset \prod_i \psi_i \, e^{-V^{(L)} + i n \cdot \Phi(0)}$$

$$\psi_i \to \psi_i \, e^{-i q_i \lambda(x)}$$

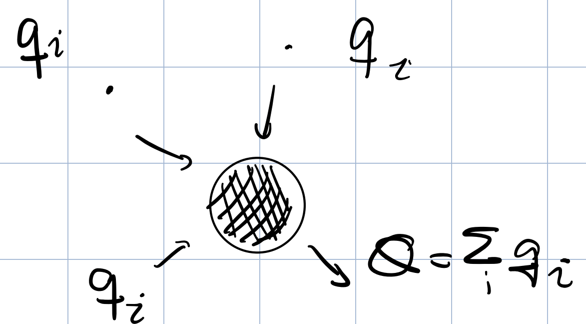
Invariant for $\sum_i q_i = k \cdot n \Rightarrow \mathbb{Z}_k$ symmetry

ii) Black Hole arguments

Consider global, continuous symmetry G (acting on particles for simplicity)

Claim: Implies tension w/ statistical interpretation of Black Hole (BH) entropy:

* Form BH of charge Q by throwing in particles of charge q_i



* Since G is global, its current does <u>not</u> couple to long-range gauge field A
  → effect of Q cannot be measured from outside

Equivalently: BH is a Schwarzschild BH

$$ds^2 = -f(r)\,dt^2 + f(r)^{-1}\,dr^2 + r^2\,d\Omega_2$$

$$f(r) = 1 - \frac{2\,G_N \cdot M}{r}$$

only depends on M, not Q (no hair theorem)

* In particular:

Horizon at $r_s = 2 G_N M$

area $A = \pi r_s^2$

Bekenstein-Hawking entropy : $S_{BH} = \frac{A}{4 G_N}$

is independent of Q

* Statistical interpretation

$$S_{BH} \sim \log(\Omega_{micro})$$ compatible w/ macroscopic solution

$\downarrow$

contains no information on Q, but same time can form many black holes of different Q & same M $\longleftrightarrow$ many different microstates

I.e. Infinite ignorance about microstates (charges q_i) should lead to increase of S_{BH} w/ Q, but does not.

* Resolved if symmetry G is gauged:

Then BH is of Reissner-Nordström type

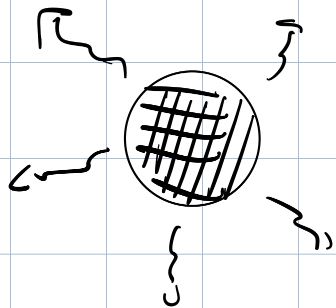
$$f(r) = 1 - \frac{2 M G_N}{r} + \frac{2 g^2 Q^2 G_N}{r^2}$$ g: U(1) coupling

$\Rightarrow r_+, r_-$ & $A(r_+) = A(M, Q)$

$\Rightarrow$ entropy $S(M,Q)$ keeps track of charge

Similar version of same argument involves "stable remnants":

BH of (global) charge Q $\rightarrow$ Hawking radiation to lose mass, but at constant Q since horizon geometry insensitive to Q



net charge loss $= 0$

At end of evaporation process:

"remnant" at $M_r = X \cdot M_{pl.}$ $X = \mathcal{O}(1)$ (?)

& charge Q

stable since for $Q \gg 1$ not enough light charged particles available to decay into

(can argue that "Bremsstrahlung" cannot do it)

$\Rightarrow$ ∞ many "species of remnants" (of different charges)

$\Rightarrow$ Speculated that such remnants are problematic

* If run in loops, lower cutoff Λ_s (species scale)

$$\frac{\Lambda_s}{M_{pl}} = \frac{1}{N_s^{\frac{1}{d-2}}}$$

N_s: # of species (below Λ_s)

scale above which no local or weakly coupled description possible

$\rightarrow N_s \rightarrow \infty \Rightarrow \Lambda_s \rightarrow 0$ (but if remnants sit at M_{pl} less clear)

* Covariant entropy bound

$$S(A) \leq \frac{A(e)}{4 G_N}$$

for remnant of size $X \cdot L_p$: $S(A) \leq \pi \cdot X^2$

$\rightarrow$ clash w/ idea of having ∞ many

<u>Note</u>: Arguments not directly applicable to discrete symmetries

Maximal order of non-gauged $\mathbb{Z}_k$:

$$\Omega_{micro} \sim k \leq e^{S} \sim e^{\pi X^2}$$

In holography / AdS gravity also discrete case can be covered.